KUTZTOWN AREA SCHOOL DISTRICT

*Maximize potential, embrace change, create the future*

# Kutztown Area School District Curriculum (Unit Map)

*Grade 12 Honors Calculus*

*Written by Sallie Sandler*

**Course Description:** This course is for students who have demonstrated exceptional achievement in prior mathematics courses and have met the established criteria for Honors/AP selection as stated in this course selection guide. The curriculum in this course is modeled after that of Advanced Placement Calculus. The primary difference between the courses lies in pacing and depth. Additionally, some topics covered in AP Calculus may be omitted in this course. However, both differential and integral calculus will be studied in sufficient depth to provide a solid foundation for college level work.

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| Unit #/Title | 1/Prerequisites for Calculus | Time Frame | 6 Weeks |
|---|---|---|---|

## Standards

**Standards of Mathematical Practices**

- Use appropriate tools strategically.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

**CC.2.2.HS.D.2** Write expressions in equivalent forms to solve problems.
**CC.2.2.HS.D.4** Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs.
**CC.2.2.HS.D.6** Extend the knowledge of rational functions to rewrite in equivalent forms.
**CC.2.2.HS.D.10** Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
**CC.2.2.HS.C.1** Use the concept and notation of functions to interpret and apply them in terms of their context.
**CC.2.2.HS.C.2** Graph and analyze functions and use their properties to make connections between the different representations.
**CC.2.2.HS.C.3** Write functions or sequences that model relationships between two quantities.
**CC.2.2.HS.C.6** Interpret functions in terms of the situations they model.
**CC.2.2.HS.C.7** Apply radian measure of an angle and the unit circle to analyze the trigonometric functions.
**CC.2.2.HS.C.8** Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs.
**CC.2.3.HS.A.14** Apply geometric concepts to model and solve real world problems.

## Big Ideas

- Algebraic concepts require representing, transitioning between, and manipulating situations numerically, symbolically, graphically, and contextually.
- Mathematical processes require the use of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas.

## Essential Questions

- What prerequisite skills are most important and necessary for learning calculus?
- What function knowledge will be essential to learning calculus?

## Content

- Point-slope form of a line
- Increment
- Domain
- Range
- Even & odd functions
- Piecewise functions
- Function composition
- Exponential growth & decay and the number e
- Parametric equations
- One-to-one functions
- Inverses
- Logarithms and their properties
- Radian measure of angles
- The Unit Circle
- Trigonometric functions
- Graphs of trigonometric functions
- Transformations of trigonometric functions

## Skills

- Write a linear equation in point-slope form
- Identify increments
- Identify domain & range of a function graphically and algebraically
- Classify a function as even, odd, neither numerically, algebraically, and graphically
- Interpret, graph and write piecewise functions
- Graph viewing skills with the graphing calculator
- Write equations to solve exponential growth & decay problems
- Graph and interpret relations and functions represented by parametric equations
- Determine whether a function is one-to-one
- Calculate and verify a function's inverse
- Solve problems involving exponential functions and logarithms
- Apply properties of logarithms to solve problems
- Convert between degree and radian measure

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|---|---|
| • Modeling with trigonometric functions<br>• Inverse trigonometric functions | • Calculate angle measures using inverse trigonometric functions<br>• Identify period, amplitude and viewing window of graphs of sinusoidal functions<br>• Draw graphs of trigonometric functions<br>• Solve equations involving trigonometric functions<br>• Write modeling equations using sinusoids |

| **Unit #/Title** | 2/Limits and Continuity | **Time Frame** | 4-5 Weeks |
|---|---|---|---|

## Standards

**Standards of Mathematical Practices**
- Reason abstractly and quantitatively.
- Look for and make use of structure.

**CC.2.2.HS.D.6** Extend the knowledge of rational functions to rewrite in equivalent forms.
**C.2.2.HS.D.10** Represent, solve, and interpret equations algebraically and graphically.
**CC.2.2.HS.C.1** Use the concept and notation of functions to interpret and apply them in terms of their context.
**CC.2.2.HS.C.2** Graph and analyze functions and use their properties to make connections between the different representations.
**CC.2.2.HS.C.6** Interpret functions in terms of the situations they model.

| **Big Ideas** | **Essential Questions** |
|---|---|
| • Mathematical processes require the use of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas. | • What is a limit?<br>• What constitutes continuity?<br>• What is an average rate of change and what does it represent?<br>• What is an instantaneous rate of change and what does it represent? |
| Content | Skills |
| • Definition of a limit<br>• Properties of limits<br>• Sandwich theorem<br>• Continuity test<br>• Properties of continuous functions<br>• Average and instantaneous rates of change<br>• Tangent and normal line equations | • Calculate limits algebraically & graphically<br>• Recognize when limits do not exist<br>• Identify horizontal & vertical asymptotes and analyze graph behavior near asymptotes<br>• Use end behavior models to find limits as x approaches $\pm\infty$<br>• Identify types of discontinuity and locate points of discontinuity<br>• Identify when a function is continuous at a point<br>• Calculate average rates of change<br>• Write equations of tangent and normal lines to a curve<br>• Calculate slopes of curves |

| Unit #/Title | 3/Derivatives | Time Frame | 8-9 Weeks |
|---|---|---|---|

## Standards

**Standards of Mathematical Practices**

- Model with mathematics.
- Use appropriate tools strategically.
- Look for and make use of structure.

**CC.2.1.HS.F.3** Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
**CC.2.1.HS.F.4** Use units as a way to understand problems and to guide the solution of multi–step problems.
**CC.2.2.HS.D.1** Interpret the structure of expressions to represent a quantity in terms of its context.
**CC.2.2.HS.D.6** Extend the knowledge of rational functions to rewrite in equivalent forms.
**CC.2.2.HS.D.8** Apply inverse operations to solve equations or formulas for a given variable.
**CC.2.2.HS.C.1** Use the concept and notation of functions to interpret and apply them in terms of their context.
**CC.2.2.HS.C.6** Interpret functions in terms of the situations they model.

## Big Ideas

- Algebraic concepts require representing, transitioning between, and manipulating situations numerically, symbolically, graphically, and contextually.
- Mathematical processes require the use of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas.

## Essential Questions

- What is a derivative and what does it represent?
- What is differentiability?
- How do we find the derivatives of relations and functions?
- How are derivatives applied in real world physical, scientific, and economic situations?

## Content

- Derivative of a function (numeric, algebraic, graphic)
- Derivative notation and terminology
- One-sided derivatives
- Differentiability (local linearity)
- Differentiability/continuity connection
- Rules for derivatives
- Higher order derivatives
- Applications of derivatives
- Chain rule
- Implicit differentiation
- Logarithmic differentiation

## Skills

- Calculate derivatives
- Graph derivatives
- Recognize when a function is not differentiable
- Apply derivatives to physical situations
- Calculate higher order derivatives
- Apply differentiation techniques

| Unit #/Title | 4/Applications of Derivatives | Time Frame | 7-8 Weeks |
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## Standards

**Standards of Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Construct viable arguments and critique the reasoning of others.
3. Model with mathematics.

**CC.2.1.HS.F.3** Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
**CC.2.1.HS.F.** Use units as a way to understand problems and to guide the solution of multi–step problems.
**CC.2.2.HS.D.1** Interpret the structure of expressions to represent a quantity in terms of its context.
**CC.2.2.HS.D.2** Write expressions in equivalent forms to solve problems.
**CC.2.2.HS.D.4** Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs.
**C.2.2.HS.D.6** Extend the knowledge of rational functions to rewrite in equivalent forms.
**CC.2.2.HS.D.8** Apply inverse operations to solve equations or formulas for a given variable.
**CC.2.2.HS.D.10** Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically
**CC.2.2.HS.C.1** Use the concept and notation of functions to interpret and apply them in terms of their context.
**CC.2.2.HS.C.3** Write functions or sequences that model relationships between two quantities.
**CC.2.2.HS.C.6** Interpret functions in terms of the situations they model.
**CC.2.3.HS.A.6** Verify and apply theorems involving similarity as they relate to plane figures.
**CC.2.3.HS.A.7** Apply trigonometric ratios to solve problems involving right triangles.
**CC.2.3.HS.A.12** Explain volume formulas and use them to solve problems.
**CC.2.3.HS.A.13** Analyze relationships between two–dimensional and three–dimensional objects.
**CC.2.3.HS.A.14** Apply geometric concepts to model and solve real world problems.

| Big Ideas | Essential Questions |
|---|---|
| • Algebraic concepts require representing, transitioning between, and manipulating situations numerically, symbolically, graphically, and contextually.<br>• Geometry requires the analysis of characteristics and properties of two- and three-dimensional shapes, describing spatial relationships, and using geometric models to solve problems.<br>• Mathematical processes require the use of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas. | • What does the derivative tell us about a function and its graph?<br>• How can a tangent line be used to approximate a function?<br>• How can the derivative be used to optimize real-world situations?<br>• How can the derivative be helpful in determining rates of change that are dependent upon one another? |
| **Content** | **Skills** |
| • Extreme values<br>• Extreme value theorem<br>• Critical point<br>• Mean value theorem<br>• Increasing, decreasing, constant<br>• Antiderivative | • Use the first derivative test for extrema<br>• Use the second derivative test for concavity<br>• Use critical points to find extrema<br>• Sketch curves based on results of first and second derivative test |

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|---|---|
| • Concavity<br>• Point of inflection<br>• Linearization<br>• Newton's method<br>• Differential<br>• Related rates | • Apply the Mean Value Theorem to real-world situations, both graphically and physically<br>• Calculate antiderivatives<br>• Solve optimization problems<br>• Estimate change with differentials<br>• Use linearization as an approximation method<br>• Apply Newton's Method to find roots of a polynomial<br>• Solve related rates problems |

| **Unit #/Title** | 5/The Definite Integral | **Time Frame** | 5-6 Weeks |
|---|---|---|---|

## Standards

**Standards of Mathematical Practices**

- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make use of structure.

**CC.2.1.HS.F.3** Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
**CC.2.1.HS.F.4** Use units as a way to understand problems and to guide the solution of multi–step problems.
**CC.2.2.HS.D.1** Interpret the structure of expressions to represent a quantity in terms of its context.
**CC.2.2.HS.C.1** Use the concept and notation of functions to interpret and apply them in terms of their context.
**CC.2.2.HS.C.4** Interpret the effects transformations have on functions and find the inverses of functions.
**CC.2.2.HS.C.5** Construct and compare linear, quadratic, and exponential models to solve problems.
**CC.2.3.HS.A.14** Apply geometric concepts to model and solve real world problems.
**CC.2.4.HS.B.2** Summarize, represent, and interpret data on two categorical and quantitative variables.

| Big Ideas | Essential Questions |
|---|---|
| • Algebraic concepts require representing, transitioning between, and manipulating situations numerically, symbolically, graphically, and contextually.<br>• Data analysis requires choosing, collecting, organizing, displaying, interpreting, and analyzing data in order to understand, model, and solve problems.<br>• Mathematical processes require the use of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas. | • How can we accumulate change over an interval to produce a function (the integral)?<br>• How can we estimate areas under a curve? |

| Content | Skills |
| --- | --- |
| • Rectangular approximation methods<br>• Approximation methods applied to volumes<br>• Riemann sums<br>• Definite integrals<br>• Integral notation & terminology<br>• Integral/area connection<br>• Fundamental theorem of calculus (Part I & II)<br>• Average function value<br>• Trapezoidal rule<br>• Simpson's rule | • Calculate left-endpoint, right-endpoint, and midpoint rectangular approximations<br>• Calculate volumes using rectangular approximations<br>• Calculate antiderivatives<br>• Calculate definite integrals using known area formulas<br>• Calculate definite integrals<br>• Calculate average function value and use it to determine when a function takes on its average value<br>• Apply the fundamental theorem of calculus to create accumulation functions<br>• Calculate trapezoidal approximations<br>• Apply Simpson's Rule to calculate parabolic approximations for polynomials |